KINEMATICS IN TWO DIMENSION

- **Projectile Motion**
  - Projectile motion is the motion of an object fired at an angle $\theta$ with the horizontal.
  - This motion can be discussed by analyzing the horizontal components of the object’s motion independently of the vertical component of motion.

\[
v_{0x} = v_0 \cos \theta
\]

\[
v_{0y} = v_0 \sin \theta
\]

- If air resistance is negligible, then the horizontal component of motion does not change $\Rightarrow$ constant velocity motion
  - The following equation describes the horizontal motion of a projectile.

\[
x = v_x t
\]
KINEMATICS IN TWO DIMENSION...

• Projectile Motion...
  – The vertical component of motion is affected by gravity ⇒ uniform constant acceleration motion
  – The motion is then described by the equations for an object in free fall.
  – The following equations are used to describe the vertical motion of a projectile.

\[
\begin{align*}
y &= v_{0y} t + \frac{1}{2} (-g) t^2 \\
v_y &= v_{0y} + (-g) t
\end{align*}
\]

⇒ \( v_y^2 = v_{0y}^2 + 2(-g)y \)

Problem 1

• In a particular laboratory experiment, a spring gun placed on a table fires a steal ball horizontally outward. A student determines that the ball starts 1.0 m above the floor and travels 2.7 m horizontally before it strikes the floor. Determine the
  – a) time that the ball is in the air and
  – b) initial velocity of the ball.

\[
x = 2.7m \\
y = -1.0m \\
v_{0x} = v_0 \cos \theta = v_0 \\
v_{0y} = v_0 \sin \theta = 0
\]

a) \( y = v_{0y} t + \frac{1}{2} (-g) t^2 \) ⇒ \( t = 0.45s \)

b) \( x = v_{0x} t \) ⇒ \( v_{0x} = v_0 = 6.0m/s \)
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- **Problem 2**
  - A spring gun placed on a table fires a steel ball at a 45° angle above the horizontal. The ball leaves the muzzle of the gun 1.1 m above the floor and travels 4.6 m horizontally. Determine the
  - a) total time that the ball is in the air.

\[
\begin{align*}
\theta &= 45^\circ \\
y &= -1.1 m
\end{align*}
\]

\[
v_{0x} &= v_0 \cos \theta = \frac{\sqrt{2}}{2} v_0 \\
v_{0y} &= v_0 \sin \theta = \frac{\sqrt{2}}{2} v_0
\]

\[
y &= v_{0y} t + \frac{1}{2} (-g) t^2 \\
x &= v_{0x} t
\]

\[
\begin{align*}
-1.1 &= \frac{\sqrt{2}}{2} v_0 t - 4.9 t^2 \\
4.6 &= \frac{\sqrt{2}}{2} v_0 t
\end{align*}
\]

\[
\Rightarrow t = 1.08 s
\]

- **Problem 3**
  - A projectile is fired with an initial speed of 113 m/s at an angle of 60.0° above the horizontal from the top of a cliff 49.0 m high. Determine the
  - a) time to reach maximum height,
  - b) maximum height above the base of the cliff reached by the projectile

\[
\begin{align*}
v_0 &= 113 m/s \\
g &= 9.8 m/s^2
\end{align*}
\]

\[
v_{0x} &= v_0 \cos \theta = 56.5 m/s \\
v_{0y} &= v_0 \sin \theta = 97.9 m/s
\]

\[
a) \rightarrow v_y = v_{0y} + (-g) t \\
\Rightarrow t = 10.0s
\]

\[
b) \rightarrow y = v_{0y} t + \frac{1}{2} (-g) t^2 \\
\Rightarrow y = 489 m
\]

\[
b) \rightarrow h = 489 m + 49 m \\
\Rightarrow h = 538 m
\]
KINEMATICS IN TWO DIMENSION...

• Problem 4
  – A projectile is fired with an initial speed of 113 m/s at an angle of 60.0° above the horizontal from the top of a cliff 49.0 m high. Determine the
  – a) total time it stays in the air, and
  – b) horizontal range of the projectile.

\[ v_0 = 113 \text{ m/s} \quad v_{0x} = v_0 \cos \theta = 56.5 \text{ m/s} \]
\[ \theta = 60.0° \quad v_{0y} = v_0 \sin \theta = 97.9 \text{ m/s} \]
\[ y = -49 \text{ m} \]

\[ a) \quad y = v_{0y}t + \frac{1}{2}(-g)t^2 \quad \Rightarrow t = 20.5 \text{ s} \]

\[ b) \quad x = v_{0x}t \quad \Rightarrow x = 1156 \text{ m} \]

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• Problem 5
  – A stone is thrown horizontally outward from the top of a bridge. The stone is released 19.6 m above the street below. The initial velocity of the stone is 5.0 m/s. Determine the
  – a) total time that the stone is in the air and
  – b) magnitude and direction of the velocity of the projectile “just” before it strikes the street.

\[ v_0 = 5.0 \text{ m/s} \quad v_{0x} = v_0 \cos \theta = 5.0 \text{ m/s} \]
\[ y = -90.6 \text{ m} \quad v_{0y} = v_0 \sin \theta = 0 \]
\[ g = 9.8 \text{ m/s}^2 \]

\[ a) \quad y = v_{0y}t + \frac{1}{2}(-g)t^2 \quad \Rightarrow t = 2.0 \text{ s} \]

\[ b) \quad v_y = v_{0y} + (-g)t \quad \Rightarrow v_y = -19.6 \text{ m/s} \]

\[ b) \quad v = \sqrt{v_x^2 + v_y^2} \quad \Rightarrow v = 20.0 \text{ m/s} \]

\[ b) \quad \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) \quad \Rightarrow \theta = 76° \]

The direction is 76° south of east
• Relative Velocity  
  – Relative velocity refers to the velocity of an object with respect to a particular frame of reference.  
  – The reference frame is usually specified by using Cartesian coordinates, i.e., x and y axis, relative to which the position and/or motion of an object can be determined.

\[ \vec{V}_{BS} = \vec{V}_{BW} + \vec{V}_{WS} \]

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• Relative Velocity...  
  – The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference and the relative velocity of the two reference frames are known.
**KINEMATICS IN TWO DIMENSION...**

**Problem 6**
- The current in the river is 1.0 m/s. A woman swims 300 m down stream and then back to her starting point without stopping. If she can swim 2.0 m/s in still water, determine the time required for the round trip.

**downstream**
\[ V_{PS} = V_{PW} + V_{WS} \quad \Rightarrow V_{PS} = 2.0 + 1.0 \quad \Rightarrow V_{PS} = 3.0 \text{ m/s} \]

\[ x = vt \quad \Rightarrow t = 100 \text{s} \]

**upstream**
\[ V_{PS} = V_{PW} + V_{WS} \quad \Rightarrow V_{PS} = 2.0 + (-1.0) \quad \Rightarrow V_{PS} = 1.0 \text{ m/s} \]

\[ x = vt \quad \Rightarrow t = 300 \text{s} \]

\[ \Rightarrow \text{time} = 100 + 300 = 400 \text{s} \]

**Problem 7**
- The woman in the previous problem swims perpendicular across the river to the opposite bank. If the river is 300 m wide, determine
  - a) the woman's velocity relative to the shore,
  - b) the distance swept downstream and
  - c) the time required to swim across the river.

\[ V_{PS} = V_{PW} + V_{WS} \quad \Rightarrow V_{PS} = \sqrt{(V_{PW})^2 + (V_{WS})^2} \quad \Rightarrow V_{PS} = 2.2 \text{ m/s} \]

\[ \Rightarrow \theta = \tan^{-1} \left( \frac{V_{WS}}{V_{PW}} \right) \quad \Rightarrow \theta = 27^\circ \]

\[ \tan 27^\circ = \frac{y}{300} \quad \Rightarrow y = 152.8 \]

\[ x = v_x t \quad \Rightarrow 300 = 2t \quad \Rightarrow t = 150 \text{s} \]